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A new method for solving conformable fractional coupled viscous Burgers' equation

Hadi Rezazadeh[1], Mostafa Eslami[2], Sanaz Abedin Taleghani[2], Alper Korkmaz [3*]
[1] Faculty of Engineering Technology, Amol University of Special Modern Technologies, Amol, Iran
[2] Department of Mathematics, Faculty of Mathematical Sciences, University of Mazandaran, Babolsar, Iran
[3] Department of Mathematics, ÇAKU, Çankırı, Turkey
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Abstract: In this paper, the combined fractional Laplace transform and new Adomian decomposition method (FLTNADM) is employed for solving conformable fractional coupled Burgers equation. Because of difficulty and complex procedure of Adomian polynomials calculations, He polynomials based on homotopy perturbation method instead of Adomian polynomials, are used. The results obtained are in good agreement with the exact solution. These results show that the technique introduced here is accurate and easy to apply with less computational work than other methods.

Keywords: Conformable fractional Coupled Burgers equation; Fractional Laplace transform method, New Adomian decomposition method, He polynomials.

1 Introduction

In literature there are many definitions on fractional derivatives but the most frequently used are as below.
i) Riemann-Liouville fractional derivative is defined by [1]

$$D_t^{\alpha} f(t) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_0^t (t-\varepsilon)^{-\alpha} f(\varepsilon) d\varepsilon, \qquad 0 \leq \alpha < 1. \tag{1}$$

ii) Caputo fractional derivative is defined by [1]

$$D_t^{\alpha} f(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-\varepsilon)^{-\alpha} f'(\varepsilon) d\varepsilon, \qquad 0 \leq \alpha < 1. \tag{2}$$

iii) Jumaries fractional derivative is defined by [2]

$$D_t^{\alpha} f(t) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dx} \int_0^t (t-\varepsilon)^{-\alpha} (f(\varepsilon) - f(0)) d\varepsilon, \qquad 0 < \alpha < 1. \tag{3}$$

iv) He's fractional derivative is defined by [3]

$$D_t^{\alpha} f(t) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_0^t (t-\varepsilon)^{-\alpha} (f_0(\varepsilon) - f(\varepsilon)) d\varepsilon, \qquad 0 < \alpha < 1. \tag{4}$$

where $f_0(\varepsilon)$ is the solution of its continuous partner of the problem with the same initial condition of the fractal partner.

$$D_t^{\alpha,\beta} f(t) = I_t^{\beta(1-\alpha)} \frac{d}{dt} I_t^{(1-\beta)(1-\alpha)} f(t), \quad t> 0, \quad 0 < \alpha < 1,\ 0 \leq \beta \leq 1, \tag{5}$$

* **Corresponding author**. *E-mail address*: korkmazalper@yandex.com.tr (Alper Korkmaz)

Most of the above fractional operators are defined via the fractional integrals with singular kernels which are due to their nonlocal structures. In addition, most of the nonlocal fractional derivatives don't obey the basic chain, quotient and product rules. Recently, to overcome these and other difficulties, Khalil et al. [5] introduced a new well-behaved definition of local fractional (non-integer order) derivative, called the conformable fractional derivative. The conformable fractional derivative is theoretically very easier to handle. Recently a different study appeared in literature on conformable fractional derivative. Dazhi Zhao and Maokang Luo generalized the conformable fractional derivative and give the physical interpretation of generalized conformable derivative [6]. The conformable calculus is very fascinating and is gaining an interest-see [7]-[16] and reference therein.

Mathematical models of basic flow equations describing unsteady transport problems consist of system of nonlinear parabolic and hyperbolic partial differential equations. The coupled Burgers' equations form an important class of such partial differential equations. These equations occur in a large number of physical problems such as the phenomena of turbulence, flow through a shock wave traveling in a viscous fluid, sedimentation of two inds of particles in fluid suspensions under the effect of gravity [17, 18]. Various methods have been introduced for numerical and solution of the coupled Burgers' equations. An application of mesh free interpolation method for the numerical solution of the coupled nonlinear partial differential equation is proposed in [19]. Khater et al. [20] have obtained approximate solution of the viscous coupled Burgers' equation using cubic-spline collocation method. The equation has been solved by Deghan et al. [21] using a Pade technique and Rashid et al. [22] have used Fourier Pseudo spectral method to find numerical solution of the equation. Variational iteration method has been presented for solving the coupled viscous Burgers' equation by Adbou and Soliman [23]. The exact solution of the equation has been obtained by Kaya [24] using Adomian Decomposition method and Soliman [25] presented modified extended tanh-function method to obtain its exact solution. Biazar and Ghazvini [26] proposed the homotopy perturbation method to obtain the exact solution of nonlinear Burgers' equation. Aminikhah [27] proposed a new method based on combination of the Laplace transform and new homotopy perturbation method (NHPM) first time and used it to obtain closed form solutions of coupled viscous Burgers' equation. This method has been used for solving some equations successfully [28, 29]. The main disadvantage of the Adomian method is the complex and difficult procedure for calculation the Adomian polynomials. Ghorbani [30], introduced He polynomials based on homotopy perturbation method to calculate Adomian polynomials, making the solution procedure in Adomian method remarkable simple and straightforward.

In this paper, we use the fractional Laplace transform and a new Adomian decomposition method (FLTNADM) to obtain approximate solutions with high accuracy in for homogeneous and inhomogeneous conformable fractional coupled Burgers' equations.

Rest of the paper is organized as follows. In Section 2, we will describe the conformable fractional derivative. In Section 3, the flow analysis and mathematical formulation are presented. Section 4, includes analysis of the new technique for conformable fractional coupled Burgers' equations. In Section 5, we apply the proposed method to different types of problems related to the conformable fractional coupled Burgers' equation and finally in Section 6, we summarize the results.

2 Conformable fractional derivative

Conformable fractional derivative of order α is defined by the following definition.

Definition 1 *Let $f:(0,\infty)\to,\mathbb{R}$, then, the conformable fractional derivative of f of order α is defined as [19]*

$$D_t^{\alpha} f(t)=\lim_{\varepsilon\to 0}\frac{f(t+\varepsilon t^{1-\alpha})-f(t)}{\varepsilon}, \tag{6}$$

for all $t>0,\ \alpha\in(0,1)$.

If f is $\alpha-$differentiable in some $(0,a),\ a>0$, and $\lim_{t\to 0^+} D_t^{\alpha} f(t)$ exists, then by definition

$$D_t^{\alpha} f(0)=\lim_{t\to 0^+} D_t^{\alpha} f(t). \tag{7}$$

This new definition of the derivative has been used to solve various problems arising in different fields of science and engineering [31–33]. The new definition satisfies the properties which given in the following theorem.

Theorem 1
- $D_t^{\alpha}(af(t)+bg(t))=a\ D_t^{\alpha} f(t)+b\ D_t^{\alpha} g(t),\ \text{for all } a,b\in\mathbb{R}.$

- $D_t^{\alpha}(t^{\mu}) = \mu\ t^{\mu-\alpha},$ for all $\mu \in \mathbb{R}$.
- $D_t^{\alpha} f(t)g(t) = f(t)\ D_t^{\alpha} g(t) + g(t)\ D_t^{\alpha} f(t)$.
- $D_t^{\alpha} \frac{f(t)}{g(t)} = \frac{g(t)\ D_t^{\alpha} f(t) - f(t)\ D_t^{\alpha} g(t)}{g(t)^2}$.

In [21] T. Abdeljawad established the chain rule for conformable fractional derivatives as following theorem.

Theorem 2 *Let $f : (0, \infty) \to \mathbb{R}$, be a function such that f is differentiable and also $\alpha-$differentiable. Let g be a function defined in the range of f and also differentiable; then, one has the following rule*

$$D_t^{\alpha}(fog)(t) = D_t^{\alpha} f\left(g(t)\right) D_t^{\alpha} g(t)\ g(t)^{\alpha-1}. \tag{8}$$

$$e^{\frac{1}{\alpha}t^{\alpha}} = \sum_{k=0}^{\infty} \frac{t^{k\alpha}}{\alpha^k k!}.$$

Now, we list here the fractional derivatives of certain functions [19]

- $D_t^{\alpha}(e^{\frac{1}{\alpha}t^{\alpha}}) = e^{\frac{1}{\alpha}t^{\alpha}}$,
- $D_t^{\alpha}(\sin \frac{1}{\alpha}t^{\alpha}) = \cos \frac{1}{\alpha}t^{\alpha}$,
- $D_t^{\alpha}(\cos \frac{1}{\alpha}t^{\alpha}) = -\sin \frac{1}{\alpha}t^{\alpha}$,
- $D_t^{\alpha}(\frac{1}{\alpha}t^{\alpha}) = 1$.

On letting $\alpha = 1$ in these derivatives, we get the corresponding ordinary derivatives.

Definition 2

$${}_tI_a^{\alpha} f(t) = \int_a^t \frac{f(x)}{x^{1-\alpha}} dx, \tag{9}$$

if the Riemann improper integral exists.

Theorem 3 ***(Integration by parts)*** *[21] Let $f, g : [0, b] \to R$ be two functions such that fg is differentiable. Then*

$$\int_0^b f(t)\ D_t^{\alpha} g)(t)\ d\alpha(t) = f(t)g(t)|_0^b - \int_0^b g(t)\ D_t^{\alpha} f(t)\ d\alpha(t), \tag{10}$$

where $d\alpha(t) = t^{\alpha-1}dt$.

It is interesting to observe that the $\alpha-$fractional derivative and the $\alpha-$fractional integral are inverse of each other as given in [19].

Theorem 4 ***(Inverse property)*** *Let $\alpha \in (0, 1]$ and f be a continuous function such that ${}_tI_0^{\alpha} f$ exists. Then*

$$D_t^{\alpha}({}_tI_0^{\alpha} f)(t) = f(t), \qquad for\ \ t \geq 0.$$

Definition 3

$$\mathcal{L}_{\alpha}\{f(t)\} = F_{\alpha}(s) = \int_0^{\infty} e^{-s\frac{t^{\alpha}}{\alpha}} f(t)\ d\mu(t), \tag{11}$$

where $d\alpha(t) = t^{\alpha-1}dt$.

Fractional Laplace transform for certain functions are presented as follows [21]

- $\mathcal{L}_{\alpha}\{1\} = \frac{1}{s}, \quad s > 0.$
- $\mathcal{L}_{\alpha}\{\frac{t^{\alpha}}{\alpha}\} = \frac{1}{s^2}, \quad s > 0.$
- $\mathcal{L}_{\alpha}\{e^{\frac{t^{\alpha}}{\alpha}}\} = \frac{1}{s-1}, \quad s > 1.$
- $\mathcal{L}_{\alpha}\{sin\frac{1}{\alpha}t^{\alpha}\} = \frac{1}{s^2+1}, \quad s > 1.$
- $\mathcal{L}_{\alpha}\{cos\frac{1}{\alpha}t^{\alpha}\} = \frac{s}{s^2+1}, \quad s > 1.$

Furthermore, using the properties of the fractional exponential function and integration by parts, we have

$$\mathcal{L}_{\alpha}\{D_t^{\alpha}\left(f\right)(t)\} = s\ F_{\alpha}(s) - f(0). \tag{12}$$

3 Basic Idea of the Method

In this section, we illustrate basic idea of the proposed approach. Consider the following nonlinear conformable fractional equation

$$D_x^{\alpha} u + N(u) = f(x), \tag{13}$$

$$D_x^{\alpha} U - \sum_{n=0}^{\infty} a_n \mu_n(x) + p \sum_{n=0}^{\infty} a_n \mu_n(x) + N(U) = f(x), \tag{14}$$

can be decomposed into an infinite series of polynomials given by

$$N(U) = \sum\nolimits_{n=0}^{\infty} H_n(U_0, U_1, ..., U_n), \tag{15}$$

where $H_n(u_0, u_1, ..., u_n)$ are called He polynomials and are defined by

$$H_n(U_0, U_1, ..., U_n) = (\frac{1}{n!}) \left[\frac{d^n}{dp^n} N(\sum_{k=0}^{n} p^k U_k) \right]_{p=0}, \quad n = 0, 1, 2, ... \tag{16}$$

Alternatively, approximation solution of (14) can be expressed as follows

$$D_x^{\alpha} U - \sum_{n=0}^{\infty} a_n \mu_n(x) + p \sum_{n=0}^{\infty} a_n \mu_n(x) + \sum_{n=0}^{\infty} H_n(U_0, U_1, ..., U_n) = f(x). \tag{17}$$

By applying fractional Laplace transform on both sides of (17), we have

$$\mathcal{L}_{\alpha} \left\{ D_x^{\alpha} U - \sum_{n=0}^{\infty} a_n \mu_n(x) + p \sum_{n=0}^{\infty} a_n \mu_n(x) + \sum_{n=0}^{\infty} H_n(U_0, U_1, ..., U_n) - f(x) \right\} = 0, \tag{18}$$

using the differential property of fractional Laplace transform we have

$$s\mathcal{L}_{\alpha}\{U\} - U(0) = \mathcal{L}_{\alpha} \left\{ \sum_{n=0}^{\infty} a_n \mu_n(x) - p \sum_{n=0}^{\infty} a_n \mu_n(x) - \sum_{n=0}^{\infty} H_n(U_0, U_1, ..., U_n) + f(x) \right\}, \tag{19}$$

or

$$\mathcal{L}_{\alpha}\{U\} = \frac{1}{s} \left\{ U(0) + \mathcal{L}_{\alpha}\{\sum_{n=0}^{\infty} a_n \mu_n(x) - p \sum_{n=0}^{\infty} a_n \mu_n(x) - \sum_{n=0}^{\infty} H_n(U_0, U_1, ..., U_n) + f(x)\} \right\}. \tag{20}$$

By applying inverse fractional Laplace transform on both sides of (20), we have

$$\mathcal{L}_{\alpha}^{-1} \left\{ \frac{1}{s}\{U(0) + \mathcal{L}_{\alpha}\{\sum_{n=0}^{\infty} a_n \mu_n(x) - p \sum_{n=0}^{\infty} a_n \mu_n(x) - \sum_{n=0}^{\infty} H_n(U_0, U_1, ..., U_n) + f(x)\}\} \right\}, \tag{21}$$

according to (21), we define

$$\begin{aligned} U_0(x) &= \mathcal{L}_{\alpha}^{-1} \left\{ \frac{1}{s} U(0) + \frac{1}{s} \mathcal{L}_{\alpha}\{\sum_{n=0}^{\infty} a_n \mu_n(x)\} \right\}, \\ U_1(x) &= \mathcal{L}_{\alpha}^{-1} \left\{ \frac{1}{s} \mathcal{L}_{\alpha}\{-p \sum_{n=0}^{\infty} a_n \mu_n(x) - H_0(U_0) + f(x)\} \right\}, \\ &\vdots \\ U_j(x) &= \mathcal{L}_{\alpha}^{-1} \left\{ -\tfrac{1}{s} \mathcal{L}_{\alpha}\{H_{j-1}(U_0, U_1, ..., U_{j-1})\} \right\}, \quad j = 2, 3, \ldots \\ U_0(x) &= \mathcal{L}_{\alpha}^{-1} \left\{ \frac{1}{s} U(0) + \frac{1}{s} \mathcal{L}_{\alpha}\{\sum_{n=0}^{\infty} a_n \mu_n(x)\} \right\}. \end{aligned} \tag{22}$$

In this method, only the first He polynomial is calculated, and the method does not need to solve the functional equation in each iteration.

4 Analysis of the Method for conformable fractional coupled Burgers'equations

According to the proposed method, we consider the following conformable fractional coupled Burgers' equations

$$\begin{array}{l} D_t^{\alpha} u - u_{xx} + \eta u u_x + \kappa (uv)_x = f(x,t), \\ D_t^{\alpha} v - v_{xx} + \eta v v_x + \gamma (uv)_x = g(x,t), \end{array} \tag{23}$$

$$u(x,0) = \phi_1(x), \; v(x,0) = \phi_2(x), \tag{24}$$

For solving this conformable fractional equation by new method, we consider the following equation

$$\begin{array}{l} D_t^{\alpha} U - \sum\limits_{n=0}^{\infty} a_n(x)\mu_n(t) + p\sum\limits_{n=0}^{\infty} a_n(x)\mu_n(t) - U_{xx} + \eta U U_x + \kappa (UV)_x = f(x,t), \\ D_t^{\alpha} V - \sum\limits_{n=0}^{\infty} b_n(x)\lambda_n(t) + p\sum\limits_{n=0}^{\infty} b_n(x)\lambda_n(t) - V_{xx} + \eta V V_x + \gamma (UV)_x = g(x,t), \end{array} \tag{25}$$

It is assumed that the unknown functions can be expressed by an infinite series in the form

$$\begin{array}{l} U = U_0 + U_1 + U_2 + \cdots, \\ V = V_0 + V_1 + V_2 + \cdots. \end{array}$$

By applying fractional Laplace transform on both sides of (25), we have

$$\begin{array}{l} \mathcal{L}_{\alpha}\left\{ D_t^{\alpha} U - \sum\limits_{n=0}^{\infty} a_n(x)\mu_n(t) + p\sum\limits_{n=0}^{\infty} a_n(x)\mu_n(t) - U_{xx} + \eta U U_x + \kappa (UV)_x - f(x,t) \right\} = 0, \\ \mathcal{L}_{\alpha}\left\{ D_t^{\alpha} V - \sum\limits_{n=0}^{\infty} b_n(x)\lambda_n(t) + p\sum\limits_{n=0}^{\infty} b_n(x)\lambda_n(t) - V_{xx} + \eta V V_x + \gamma (UV)_x - g(x,t) \right\} = 0, \end{array} \tag{26}$$

using the differential property of fractional Laplace transform we have

$$\begin{array}{l} s\mathcal{L}_{\alpha}\{U\} - U(x,0) \\ \quad = \mathcal{L}_{\alpha}\left\{ \sum\limits_{n=0}^{\infty} a_n(x)\mu_n(t) - p\sum\limits_{n=0}^{\infty} a_n(x)\mu_n(t) + U_{xx} - \eta U U_x - \kappa (UV)_x + f(x,t) \right\}, \\ s\mathcal{L}_{\alpha}\{V\} - V(x,0) \\ \quad = \mathcal{L}_{\alpha}\left\{ \sum\limits_{n=0}^{\infty} b_n(x)\lambda_n(t) - p\sum\limits_{n=0}^{\infty} b_n(x)\lambda_n(t) + V_{xx} - \eta V V_x - \gamma (UV)_x + g(x,t) \right\}, \end{array} \tag{27}$$

or

$$\begin{array}{l} \mathcal{L}_{\alpha}\{U\} = \frac{1}{s}\{U(x,0) \\ \quad +\mathcal{L}_{\alpha}\left\{ \sum\limits_{n=0}^{\infty} a_n(x)\mu_n(t) - p\sum\limits_{n=0}^{\infty} a_n(x)\mu_n(t) + U_{xx} - \eta U U_x - \kappa (UV)_x + f(x,t) \right\}\Big\}, \\ \mathcal{L}_{\alpha}\{U\} = \frac{1}{s}\{V(x,0) \\ \quad +\mathcal{L}_{\alpha}\left\{ \sum\limits_{n=0}^{\infty} b_n(x)\lambda_n(t) - p\sum\limits_{n=0}^{\infty} b_n(x)\lambda_n(t) + V_{xx} - \eta V V_x - \gamma (UV)_x + g(x,t) \right\}\Big\}. \end{array} \tag{28}$$

By applying inverse fractional Laplace transform on both sides of (28), we have

$$\begin{array}{l} U(x,t) = \mathcal{L}_{\alpha}^{-1}\left\{ \frac{1}{s}\{U(x,0) \right. \\ \quad \left. +\mathcal{L}_{\alpha}\{ \sum\limits_{n=0}^{\infty} a_n(x)\mu_n(t) - p\sum\limits_{n=0}^{\infty} a_n(x)\mu_n(t) + U_{xx} - \eta(\sum\limits_{n=0}^{\infty} A_n) - \kappa(\sum\limits_{n=0}^{\infty} C_n) + f(x,t)\}\} \right\}, \\ V(x,t) = \mathcal{L}_{\alpha}^{-1}\left\{ \frac{1}{s}\{V(x,0) \right. \\ \quad \left. +\mathcal{L}_{\alpha}\{ \sum\limits_{n=0}^{\infty} b_n(x)\lambda_n(t) - p\sum\limits_{n=0}^{\infty} b_n(x)\lambda_n(t) + V_{xx} - \eta(\sum\limits_{n=0}^{\infty} B_n) - \gamma(\sum\limits_{n=0}^{\infty} C_n) + g(x,t)\}\} \right\}, \end{array} \tag{29}$$

$$A_n\left(U_0, U_1, \ldots, U_n\right) = \left(\frac{1}{n!}\right)\left[\frac{d^n}{d\lambda^n}[N_1(\sum_{k=0}^{\infty}\lambda^k U_k)]\right]_{\lambda=0},$$

$$B_n\left(V_0,V_1,\ldots,V_n\right)=(\frac{1}{n!})\left[\frac{d^n}{d\lambda^n}[N_2(\sum_{k=0}^{\infty}\lambda^k V_k)]\right]_{\lambda=0},$$

$$C_n\left(U_0,U_1,\ldots,U_n,V_0,V_1,\ldots,V_n\right)=(\frac{1}{n!})\left[\frac{d^n}{d\lambda^n}[N_3(\sum_{k=0}^{\infty}\lambda^k U_k.\sum_{k=0}^{\infty}\lambda^k V_k)]\right]_{\lambda=0},$$

according to (29), we define

$$\begin{cases} U_0(x,t)=\mathcal{L}_\alpha^{-1}\left\{\frac{1}{s}U(x,0)+\frac{1}{s}L_\alpha\{\sum\limits_{n=0}^{\infty}a_n\mu_n(t)\}\right\}, \\ V_0(x,t)=\mathcal{L}_\alpha^{-1}\left\{\frac{1}{s}V(x,0)+\frac{1}{s}L_\alpha\{\sum\limits_{n=0}^{\infty}b_n\lambda_n(t)\}\right\}, \end{cases}$$

$$\begin{cases} U_1(x,t)=\mathcal{L}_\alpha^{-1}\left\{\frac{1}{s}(L_\alpha\{-p\sum\limits_{n=0}^{\infty}a_n\mu_n(t)+(U_0)_{xx}-\eta A_0-\kappa C_0+f(x,t)\})\right\}, \\ V_1(x,t)=\mathcal{L}_\alpha^{-1}\left\{\frac{1}{s}(L_\alpha\{-p\sum\limits_{n=0}^{\infty}b_n\lambda_n(t)+(V_0)_{xx}-\eta B_0-\gamma C_0+g(x,t)\})\right\}, \end{cases}$$

$$\vdots$$

$$\begin{cases} U_j(x,t)=\mathcal{L}_\alpha^{-1}\left\{\frac{1}{s}\left(L_\alpha\{-(U_{j-1})_{xx}-\eta A_{j-1}-\kappa C_{j-1}\}\right)\right\}, \\ V_j(x,t)=\mathcal{L}_\alpha^{-1}\left\{\frac{1}{s}\left(L_\alpha\{-(V_{j-1})_{xx}-\eta B_{j-1}-\gamma C_{j-1}\}\right)\right\}, \end{cases} \quad j=2,3,\ldots. \tag{30}$$

Now, let us determine $a_0,a_1,a_2,...$ so that $U_1=0$, then from (30) we have $U_2=U_3=...=0$. Setting $p=1$, results in the solution of equation (23) with the initial conditions as the following

$$\begin{cases} U_0(x,t)=\mathcal{L}_\alpha^{-1}\left\{\frac{1}{s}\phi_1(x)+\frac{1}{s}\mathcal{L}_\alpha\{\sum\limits_{n=0}^{\infty}a_n\mu_n(t)\}\right\}, \\ V_0(x,t)=\mathcal{L}_\alpha^{-1}\left\{\frac{1}{s}\phi_2(x)+\frac{1}{s}\mathcal{L}_\alpha\{\sum\limits_{n=0}^{\infty}b_n\lambda_n(t)\}\right\}. \end{cases}$$

5 Examples

Example 1. Consider the following homogeneous form of a conformable fractional coupled Burgers equation

$$\begin{array}{l} D_t^\alpha u-u_{xx}-2uu_x+(uv)_x=0, \\ D_t^\alpha v-v_{xx}-2vv_x+(uv)_x=0, \end{array} \tag{31}$$

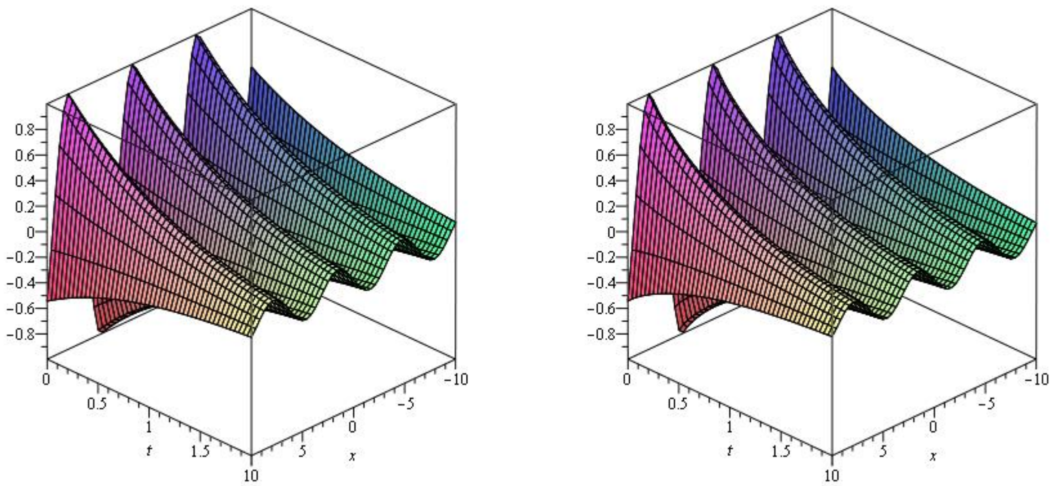
subject to the initial condition
The exact solution of the equation is as $u(x,t)=v(x,t)=e^{-\frac{1}{\alpha}t^\alpha}\sin x$.
By using FLTNADM and setting $U(x,0)=V(x,0)=\sin x$, we have

$$\begin{array}{l} U_0(x,t)=\mathcal{L}_\alpha^{-1}\left\{\frac{1}{s}(\sin(x)+\mathcal{L}_\alpha\{\sum\limits_{n=0}^{\infty}a_n(x)\mu_n(t)\})\right\} \\ V_0(x,t)=\mathcal{L}_\alpha^{-1}\left\{\frac{1}{s}(\sin(x)+\mathcal{L}_\alpha\{\sum\limits_{n=0}^{\infty}b_n(x)\lambda_n(t)\})\right\} \end{array}$$

$$\begin{cases} U_1(x,t)=\mathcal{L}_\alpha^{-1}\left\{\frac{1}{s}(\mathcal{L}_\alpha\{-p\sum\limits_{n=0}^{\infty}a_n(x)\mu_n(t)+(U_0)_{xx}-2A_0-C_0\})\right\}, \\ V_1(x,t)=\mathcal{L}_\alpha^{-1}\left\{\frac{1}{s}(\mathcal{L}_\alpha\{-p\sum\limits_{n=0}^{\infty}b_n(x)\lambda_n(t)+(V_0)_{xx}-2B_0-C_0\})\right\}, \end{cases}$$

$$a_0(x)=a_2(x)=\cdots=-\sin x, \qquad a_1(x)=a_3(x)=\cdots=\sin x,$$

$$b_0(x)=b_2(x)=\cdots=-\sin x, \qquad b_1(x)=b_3(x)=\cdots=\sin x,$$



(a) (b)

Figure 1: The surface shows the solution $u(x,y)$ and $v(x,y)$ for equation (31) when $-10 \leq x \leq 10$ and $0 < t \leq 2$: (a) $\alpha = 1$, (b) $\alpha = 0.9$

Therefore we gain the solution of equation (31) as

$$u(x,t) = U_0(x,t) = (1 - \tfrac{t^\alpha}{\alpha} + \tfrac{1}{2}\tfrac{t^{2\alpha}}{\alpha^2} - \tfrac{1}{6}\tfrac{t^{3\alpha}}{\alpha^3}^3 + \cdots)\sin x = e^{-\frac{1}{\alpha}t^\alpha}\sin x,$$

$$v(x,t) = V_0(x,t) = (1 - \tfrac{t^\alpha}{\alpha} + \tfrac{1}{2}\tfrac{t^{2\alpha}}{\alpha^2} - \tfrac{1}{6}\tfrac{t^{3\alpha}}{\alpha^3} + \cdots)\sin x = e^{-\frac{1}{\alpha}t^\alpha}\sin x,$$

which are exact solution of equation (31).

Example 2. Consider the following non-homogeneous form of a conformable fractional coupled Burgers equation

$$\begin{aligned} D_t^\alpha u - u_{xx} + uu_x + (uv)_x &= x^2 - 2\tfrac{t^\alpha}{\alpha} + (2x^3+1)\tfrac{t^{2\alpha}}{2!\alpha^2}, \\ D_t^\alpha v - v_{xx} + vv_x + (uv)_x &= \tfrac{1}{x} - \tfrac{2}{x^3}\tfrac{t^\alpha}{\alpha} - (\tfrac{1}{x^3} - 1)\tfrac{t^{2\alpha}}{2!\alpha^2}, \end{aligned} \tag{32}$$

subject to the initial condition

The exact solution of the equation is as $u(x,t) = x^2\frac{t^\alpha}{\alpha}$ and $v(x,t) = \frac{1}{x}\frac{t^\alpha}{\alpha}$.

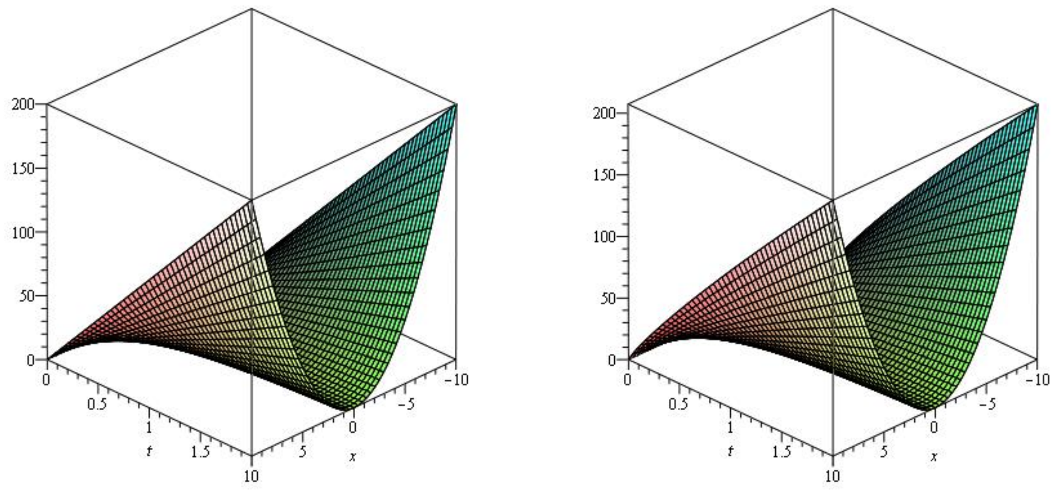
By using FLTNADM and setting $U(x,0) = V(x,0) = 0$, we have

$$\begin{cases} U_0(x,t) = \mathcal{L}_\alpha^{-1}\left\{\frac{1}{s}\left(u_0 + \mathcal{L}_\alpha\left\{U(x,0)\right\}\right)\right\}, \\ V_0(x,t) = \mathcal{L}_\alpha^{-1}\left\{\frac{1}{s}\left(v_0 + \mathcal{L}_\alpha\left\{V(x,0)\right\}\right)\right\}, \end{cases}$$

$$\begin{cases} U_1(x,t) = \mathcal{L}_\alpha^{-1}\left\{\frac{1}{s}(\mathcal{L}_\alpha\{-p\sum\limits_{n=0}^{\infty} a_n(x)\mu_n(t) + (U_0)_{xx} - A_0 - C_0 \right. \\ \qquad\qquad \left. +x^2 - 2\frac{t^\alpha}{\alpha} + (2x^3+1)\frac{t^{2\alpha}}{2!\alpha^2}\})\right\}, \\ V_1(x,t) = \mathcal{L}_\alpha^{-1}\left\{\frac{1}{s}(\mathcal{L}_\alpha\{-p\sum\limits_{n=0}^{\infty} b_n(x)\lambda_n(t) + (V_0)_{xx} - B_0 - C_0 \right. \\ \qquad\qquad \left. +\frac{1}{x} - \frac{2}{x^3}\frac{t^\alpha}{\alpha} - (\frac{1}{x^3} - 1)\frac{t^{2\alpha}}{2!\alpha^2}\})\right\}, \end{cases}$$

$$a_0(x) = x^2, \qquad a_1(x) = a_2(x) = a_3(x) = \cdots = 0,$$

$$b_0(x) = \frac{1}{x}, \qquad b_1(x) = b_2(x) = b_3(x) = \cdots = 0.$$



(a) (b)

Figure 2: The surface shows the solution $u(x, y)$ for equation (32) when $-10 \leq x \leq 10$ and $0 < t \leq 2$: (a) $\alpha = 1$, (b) $\alpha = 0.9$

Therefore we gain the solution of equation (32) as

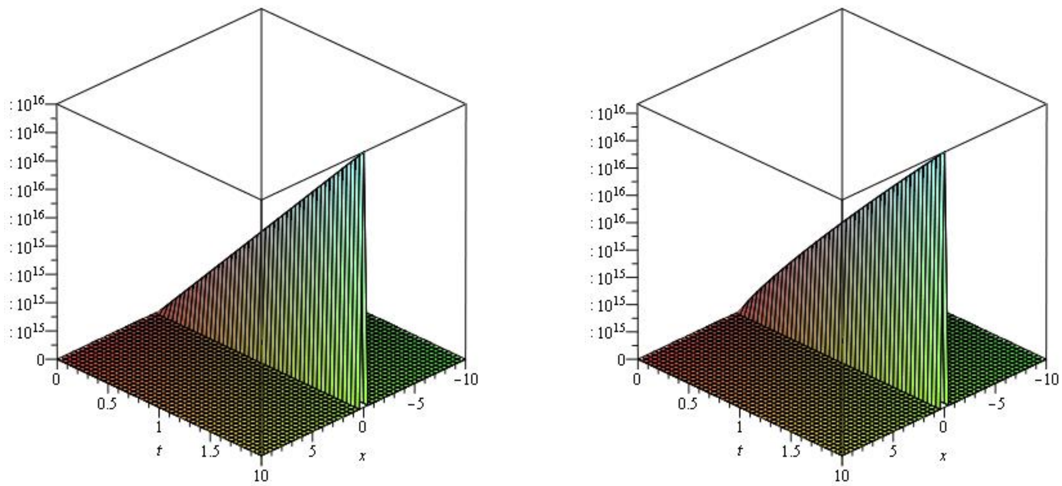
$$u(x,t) = U_0(x,t) + U_1(x,t) + \cdots = x^2 \frac{t^\alpha}{\alpha},$$

$$v(x,t) = V_0(x,t) + V_1(x,t) + \cdots = \frac{1}{x} \frac{t^\alpha}{\alpha},$$

which are exact solution of equation (32).

6 Conclusions

In this paper, the FLTNADM, a combination of fractional Laplace transform method and new Adomian decomposition method, was applied successfully to find the analytical solution of conformable fractional coupled Burgers'equations which is a conformable fractional partial differential equation. The proposed method does not require complex computations of Adomian polynomials and only use the first term of it, so the reliability of the method and reduction in the size of computations give this method a wider applicability. The obtained results show that these approaches can solve the problem effectively and can be applied to many conformable fractional nonlinear differential equations.



(a) (b)

Figure 3: The surface shows the solution $v(x, y)$ for equation (32) when $-10 \leq x \leq 10$ and $0 < t \leq 2$: (a) $\alpha = 1$, (b) $\alpha = 0.9$.

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